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Jet and Vortex Ring-Like Structures in Internal Combustion Engines: Stability Analysis and Analytical Solutions

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Abstract

Recently developed models, describing the disintegration of liquid jets and the dynamics of vortex ring-like structures at Diesel and gasoline engine-like conditions, are reviewed. The results of comparative analysis of modal and non-modal hydrodynamic instabilities of round viscous fluid jets are presented. Analytical formulae for vortex ring translational velocities, predicted by vortex ring models, are compared with the results of numerical solutions to the underlying equations and experimental data. A new approach to numerical simulation of two-phase two-dimensional flows, based on a combination of the full Lagrangian method for the dispersed phase and the vortex blob method for the carrier phase, is discussed.

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Keywords: jets; vortex rings; stability analysis; internal combustion engines.

1. Introduction

It is well known that accurate multi-dimensional modeling of the processes within Diesel and gasoline engines, and similar environments, is important for various engineering applications [1 – 5]. This modeling must take into account many complicated processes, such as heat and mass transfer, combustion and fluid dynamics. This typically leads to relatively simple models for each of these individual processes. These models are then combined together into Computational Fluid Dynamics (CFD) codes, such as the KIVA II code [6], which is an open-source non-commercial code, widely used as a basis for the analysis of sprays [7]. This code uses a Lagrangian particle tracking approach to spray modeling, which has been shown to be advantageous compared to the Eulerian approach [8, 9]. One of the most important elements of modeling Diesel and gasoline fuel injections is the accurate modeling of both jet and droplet breakup. In the case of gasoline engines, the formation of spray induced vortex ring-like structures also needs to be considered. These processes are particularly important due to

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the sensitivity of the droplet heating, evaporation [10] and combustion processes to the droplet position and size distribution. The liquid Diesel and gasoline fuel emerging from the nozzle firstly goes through a primary breakup process, where the jet breaks up into liquid sheets, ligaments and droplets. Further from the nozzle, where the fuel is dispersed in the gas phase, a secondary breakup process occurs, where large droplets breakup into smaller ones. In practical applications unified models are used to model both of these breakup processes [11]. In these unified models, the initial jet is assumed to consist of a continuous string of injected droplets with radii equal to that of the nozzle. These large droplets then undergo breakup due to the normal and tangential stresses on their surface [12]. In most CFD codes this process is described using the Taylor Analogy Breakup (TAB), WAVE breakup or stochastic breakup models. In the TAB model [13], which is the default breakup model in the KIVA II code, the breakup of fuel droplets is described using an analogy with a spring–mass system, while the WAVE model [14] uses the linear stability analysis of perturbation waves on the liquid–gas interface. In the WAVE model it is assumed that the droplets created after breakup have a predetermined size, while the TAB model takes into account the distribution of droplets by radii. The stochastic model suggested in [15] is based on the assumption that the breakup of parent droplets, at large Weber numbers, into secondary droplets, does not depend on the instantaneous size of the parent droplets. In this process the specific mechanism of atomization and the breakup length scale cannot be clearly defined, therefore the model uses stochastic approaches to model breakup rather than deterministic ones. This model has been further developed in a number of papers (see e.g. [16, 17]).

Most of the abovementioned models are primarily focused on the analysis of Diesel engine sprays, as they do not consider the formation of spray induced vortex ring-like structures, typical for gasoline engines. The effects of these structures have been generally overlooked, although they play an important role in the rate at which the liquid evaporates. To the best of our knowledge until the publication of [18] the experimentally observed vortex ring-like structures in gasoline engines and vortex ring models were never linked.

In a series of our recent papers some new approaches to modeling of spray formation processes in Diesel engine conditions and dynamics of vortex ring-like structures in gasoline engine conditions have been discussed. The main focus of this publication will be on summarizing the main results reported in these papers. Recent developments in modeling spray formation and penetration in Diesel engine-like conditions are presented and discussed in Section 2. Section 3 is focused on the comparative analysis of the results of conventional (modal) approach to the stability analysis of Diesel engine fuel jets and the results of non-modal stability analysis. New approaches to the analysis of vortex ring-like structures in gasoline engines are discussed in Section 4. The main results of the paper are summarized in Section 5.

2. Spray formation in Diesel engine-like conditions

It is generally recognized that the disintegration of jets into ligaments and droplets is caused by the combined effect of cavitation inside the nozzle and jet instabilities [19]. The theory of jet instabilities is well developed under the assumption that the jets are quasi-steady. In most practical engineering applications, however, these jets are highly transient, and this leads to systematic discrepancies between the predictions of the theory of spray formation, based on quasi-steady jet instabilities, and experimental observations [19]. In [19], one of the conventional models of spray formation was modified to achieve better agreement between the predictions of the model and the experimental data. These modifications were based on the assumption that jet acceleration leads to its stabilization in the region close to the nozzle. The model was called the modified WAVE model. This model was based on the introduction of two model constants, with no clear physical meanings, to damp the Kelvin–Helmholtz instability by jet acceleration. These constants had to be adjusted to generate agreement with experimental data.

The modified WAVE model originally suggested in [19] was further developed in [20]. In the latter paper a breakup model for analyzing the evolution of transient fuel sprays characterized by a coherent liquid core emerging from the injection nozzle, throughout the injection process, was suggested. The coherent liquid core was modeled as a liquid jet and a breakup model was formulated. The spray breakup was described using a composite model that separately addresses the disintegration of the liquid core into droplets and their further aerodynamic breakup. The jet breakup model uses the results of hydrodynamic stability theory to define the breakup length of the jet, and downstream of this point, the spray breakup process is modeled for droplets only. The composite breakup model was incorporated into the KIVA II Computational Fluid Dynamics (CFD) code and its results were compared with existing breakup models, including the classic WAVE model, the model described in [19] and in-house experimental observations of transient Diesel fuel sprays. The hydrodynamic stability results used in both the jet breakup model and the WAVE droplet breakup model were also investigated. A new velocity profile was considered for these models which consisted of a jet with a linear shear layer in the gas phase surrounding the liquid core to model the effect of a viscous gas on the breakup process. This velocity profile changed the driving instability mechanism of the jet from a surface tension driven instability for the plug flow jet with no shear layers, to an instability driven by the thickness of the shear layer. In particular, it was shown that appreciation of the shear layer instability mechanism in the composite model allows larger droplets to be predicted at jet breakup, and gives droplet sizes which are more consistent with the experimental observations. The inclusion of the shear layer into the jet velocity profile was supported by previous experimental studies, and further extended the inviscid flow theory used in the formulation of the classic WAVE breakup model.

In contrast to the model, described in [19], in the model presented in [20] only one undetermined constant, related to the primary breakup time of the jet, was used. Using the results earlier reported in [21] for the plane jet, the effect of jet acceleration on its disintegration was ignored in [20]. Both composite and modified models produced a similar level of agreement with the experimentally observed spray penetration. However, the shape of the generated spray and the droplet size distribution in the spray were predicted more accurately by the composite model compared with the modified one.

3. Modal and non-modal instabilities

To the best of our knowledge, stability of round jets has been analyzed only within the framework of the modal approach, which is based on the analysis of small disturbances to the main flow in the form of a single normal mode. The model discussed in the previous section was based solely on this analysis. At the same time, there are several examples of shear flows, where the classical stability theory fails to predict flow transition even qualitatively. For example, it is experimentally observed that the flow in a round pipe loses stability at sufficiently large Reynolds numbers, which contradicts the results of the classical hydrodynamic stability theory [22]. The discrepancy between the experiments and theoretical stability studies gave rise to so-called 'bypass' approaches, which describe the transition of shear flows without taking into account the modal (exponential) structure of disturbances. It is assumed that there are some initial disturbances to the main flow that can grow and reach the amplitude required to trigger non-linear instability or provide basic states for secondary instabilities even if all normal modes decay.

About forty years ago, it was shown theoretically that there are disturbances to all plane-parallel shear flows, which grow linearly during the limited interval of time ('lift-up' or 'vortex-tilting' mechanism). The non-modal (algebraic) growth of small disturbances can be described based on the analysis of linear differential operators, which describe the evolution of small disturbances (e.g. coupled Orr-Sommerfeld and Squire operators for normal velocity and normal vorticity [25]). These operators are non-Hermitian, which means that their system of eigenvectors (normal modes or basic solutions) is not orthogonal. It is not sufficient to consider only the eigenvector with the largest growth rate in order to describe the evolution of all small disturbances over the limited time interval, the whole spectrum should be used. Even if all eigenvectors decay (the flow is stable in the classical sense), there still can be some combinations of eigenvectors which grow significantly during a limited interval of time before they eventually decay. This non-modal or transient growth can trigger the flow transition [26]. The combinations of normal modes with the largest energy at a certain time instant (for temporal approach) or location downstream (for spatial approach) are called optimal disturbances. The non-modal or transient growth of disturbances was studied for a number of typical shear flows (e.g. [25]). It was found that the optimal disturbances to boundary layer, plane Poiseuille, Couette and pipe flows are streamwise-elongated structures similar to those found in experiments.

To the best of our knowledge, until the publication of [28] the idea of algebraic instability has not been applied to the problem of round jet break-up, while streamwise ligaments and other streamwise patterns have been observed in experiments on jet flows (e.g. [27]). In [28] the analysis of the algebraic instability of two types of round jets was performed. These are air jets in air and liquid jets in air. The analysis of the latter is expected to improve our understanding of the problem of jet break-up, as discussed in the previous section. The understanding of the jet break-up would help us to control it.

Several types of main flow velocity profiles were considered, which correspond to various distances from the orifice and various nozzle lengths. In order to perform a thorough non-modal analysis, the linear stability problem was considered in the most general way: the disturbances were assumed to be three-dimensional and non-axisymmetric. The spatially-growing disturbances were considered instead of temporally-growing ones. It allowed us to study the spatial evolution of disturbances in a downstream direction and evaluate the possible effect of non-modal growth on the break-up length of jets. The systems of three-dimensional normal modes were calculated. Disturbances which maximize the kinetic energy functional at a given position downstream were found. For all jet velocity profiles under consideration, the non-modal growth of disturbances was shown to be significant: the energy gain of optimal disturbances at a Reynolds number of one thousand was shown to be up to two orders of magnitude larger than that of the least stable single mode at the location downstream which is equivalent to about ten diameters of the jet. The non-modal growth was shown to be significant only in the case of non-axisymmetric disturbances. The strongest algebraic instability was gained by steady disturbances; the transient growth mechanism weakens relative to the exponential growth of the first mode with an increase in frequency and a decrease in Reynolds number. The streamwise velocity component of optimal disturbances to both jet types was shown to gain significantly larger growth compared to other velocity components due to the 'lift-up' mechanism. The results of this study were shown to be consistent with several experimental studies on jet break-up and theoretical studies of non-modal instabilities of a two-phase mixing layer. The strongest non-modal growth was obtained at the azimuthal mode numbers $m = 1 - 5$. It was found that there is a quadratic increase in the energy of global optimal disturbances with an increase in the Reynolds number, which is a unique feature of the 'lift-up' instability mechanism.

4. Vortex ring-like structures in gasoline engines

As mentioned in the Introduction, the effects of spray induced vortex ring-like structures in gasoline engines have been generally overlooked, although these play an important role in the rate at which the liquid fuel evaporates. The authors of [18] were perhaps the first to link the experimentally observed vortex ring-like structures in gasoline engines and vortex ring models. It was shown that vortex ring-like structures behaviour in gasoline engine-like conditions could not be adequately described in terms of the laminar vortex ring models [29]. Instead, it was suggested that the observed features of these structures were more suited to the turbulent ring model developed by Lugovtsov [30]. In the following section the most important further developments in this direction (experimental and theoretical), are summarized (see Section 4.1). Then some preliminary results focused on the application of the Full Lagrangian Method, developed by Osiptsov [31], to the analysis of vortex ring like structures are presented and discussed (see Section 4.2).

4.1 The generalized vortex ring model and its application to modeling vortex ring-like structures

The theory of vortex rings has been extensively developed and the results have been reported in several review papers and monographs. Among the most recent publications we can mention [32, 33]. The analysis of various modeling approaches is beyond the scope of this paper. In what follows, previously reported formulae describing the axial translational velocity of the vortex rings, under various approximations, are summarized, and some features of the new model are outlined.

Although some approaches to modeling of turbulent vortex rings have been suggested by Lugovtsov [30], the quantitative models have been developed mainly for laminar rings. Another assumption, useful in modeling, is the smallness of the vortex ring Reynolds number, defined as $\mathrm{Re} = \xi_0 \ell^2 / \nu$, where ξ_0 is the characteristic vorticity, ℓ is the characteristic vortex ring thickness. In the case of laminar vortex rings, $\ell = \sqrt{2\nu t}$, where t is time and ν is the kinematic viscosity of the fluid.

Using these assumptions and ignoring changes with time of the radius of the vortex ring (distance from its axis to the point where the fluid velocity is equal to zero), described by parameter R_0 , the following general equation for the normalized vortex ring axial translational velocity was obtained [29]:

$$U_x = \frac{V_x}{v_n} = \sqrt{\pi}\theta \left\{ 3\exp\left(-\frac{\theta^2}{2}\right) I_1\left(\frac{\theta^2}{2}\right) + \frac{\theta^2}{12}\, {}_2F_2\left[\frac{3}{2},\frac{3}{2};\frac{5}{2},3;-\theta^2\right] - \frac{3\theta^2}{5}\, {}_2F_2\left[\frac{3}{2},\frac{5}{2};2,\frac{7}{2};-\theta^2\right] \right\}, \quad (1)$$

where the generalized hypergeometric function ${}_2F_2\left[a_1,a_2;b_1,b_2;x\right]$ is defined as:

$${}_2F_2\left[a_1,a_2;b_1,b_2;x\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k x^k}{(b_1)_k (b_2)_k k!},$$

with the coefficients defined as:

$$(\alpha)_0 = 1;\; (\alpha)_1 = \alpha;\; , (\alpha)_k = \alpha(\alpha+1)\ldots(\alpha+k-1)\; (k \geq 2),\; \theta = \frac{R_0}{\ell},\; v_n = \frac{M}{4\pi^2 R_0^3} = \frac{\Gamma_0}{4\pi R_0}$$

$\Gamma_0 = M/\left(\pi R_0^2\right)$ is the initial circulation of the vortex ring, M is the vortex ring momentum divided by the density of the ambient fluid. The assumption of smallness of Re can be justified by the weak dependence of U_x on Re, which follows from the direct comparison of predictions of Equation (1) with the results of numerical simulations and experimental data.

In the limit of long times (small θ), Equation (1) is simplified to:

$$U_x = \tfrac{7}{30}\sqrt{\pi}\theta^3 \propto t^{-3/2}. \quad (2)$$

Although Equation (1) was originally derived, assuming that $\ell = \sqrt{2\nu t}$, it remains valid in the more general case when [34]:

$$\ell = a\, t^b, \quad (3)$$

where a and b are constants. The model based on Equation (3) was called the generalized vortex ring model [34]. It was shown that a physically correct solution can be expected when [34]

$$1/4 \leq b \leq 1/2. \quad (4)$$

When $a = \sqrt{2\nu}$ and b = 1/2, ℓ predicted by Equation (3) reduces to the one predicted by the conventional laminar vortex ring model. When b = 1/4, then some predictions of the model are similar to those which follow from the turbulent vortex rings model [30]. From this point of view, the generalized model is expected to incorporate both laminar and turbulent vortex ring models, developed earlier. In the limit of large times, the model predicts the following value of U_x :

$$U_x = \tfrac{7}{30}\sqrt{\pi} R_0^3 a^{-3} t^{-3b}. \quad (5)$$

Equation (5) is a straightforward generalization of Equation (2). Note that the velocity U_x , predicted by Equation (5), is different from the velocities of the fluid in the region of maximal vorticity, $U_{\omega x} = V_{\omega x} / v_n$. In the case of small θ (large t), they are linked by the following equation:

$$U_{\omega x} = U_x + 2\pi\theta^2 \int_0^{\infty} \mu \,\mathrm{erfc}\left(\frac{\mu}{\sqrt{2}}\right) J_1(\theta\mu) J_0(\mu)\, \mathrm{d}\mu, \quad (6)$$

where

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-t^2)\mathrm{d}t,$$

and J_1 and J_0 are Bessel functions of the first and zero order respectively. U_x is defined by Equation (5). Note that for sufficiently large μ_0, the contribution of $\mu > \mu_0$ in the integral in Equation (6) can be ignored. On the other hand, for $\theta << \mu_0^{-1}$, we can write

$$J_1(\theta\mu) = \tfrac{1}{2}\theta\mu, \tag{7}$$

Combining Equations (5) – (7) we obtain

$$U_{\omega x} = \left[\tfrac{7}{30}\sqrt{\pi} + \pi \int_0^{\infty} \mu^2 \,\mathrm{erfc}(\frac{\mu}{\sqrt{2}}) J_0(\mu)\mathrm{d}\mu \right] \theta^3. \tag{8}$$

Comparing Equations (5) and (8), we can see that both U_x and $U_{\omega x}$ are proportional to $\theta^3 \propto t^{-3b}$, that is

$$U_{\omega x} \propto t^{-3b}. \tag{9}$$

As in the case of the conventional vortex ring model, the radial component of its velocity, both in the region where the velocity is close to zero and in the region where the vorticity is maximal, predicted by the generalized model, is equal to zero.

The above results were applied to the analysis of vortex ring-like structures in gasoline engine-like conditions [35]. The focus was on (low-pressure) port-injected (PFI) and (high-pressure) direct injection (G-DI) fuel sprays. In both cases, fuel was injected into an ambient pressure and temperature, quiescent, optical chamber. High-speed photography was used to show that both fuel sprays comprised complex, vortex ring-like structures that exhibit a range of spatial scales that occurred over a broad range of timescales. An optimized, classical, forward scatter PDA was used to measure the spray droplet diameters and velocities over a fine measurement grid. An analysis of the temporal evolution of droplet axial velocities and diameters showed that both sprays comprise of the classical three phases; initial unsteady, main quasi-steady and exponential trailing phase. The results showed that the timing and main features of these phases are highly dependent upon the location of the probe volume in the spray; the relative phasing of these periods was different between the injectors studied. The vortex ring-like structures were observed mainly during the decaying phase of spray development. The mean vorticity magnitude was calculated within consecutive time intervals where vortex ring-like structures could be identified. In the port injected spray, the range was 13.75 to 14.25 ms after start of injection (SOI). In the gasoline direct injection (G-DI) case, the range was 3.75 to 4.75 ms after SOI. The maximum vorticity in the G-DI case was shown to be approximately 4 times greater than that computed in the PFI spray. The direct measurement of the axial and radial velocity components in the region corresponding to maximal vorticity was used. In both sprays, the radial component showed the most scatter in data and was close to zero for both the G-DI and PFI injectors. The observations in the G-DI case were shown to be consistent with the predictions of the generalized vortex ring model described earlier. The axial components of velocity were positive for all values of t and the PFI injector showed the most scatter in data. In the PFI case, the axial data was approximated as $V_{\omega x}(t) = 1239t^{-2.21}$. In the G-DI case, the axial data showed a better curve fit that was approximated as $V_{\omega x}(t) = 74.1t^{-1.57}$. In both cases, t refers to time elapsed after the start of injection. Periodic oscillations of about 1 kHz were noted in both sprays' axial and radial vortex velocity components. The results were presented by normalizing the time with respect to the initial time at which the vortex ring-like structures were first observed (t_{init}), with $\bar{t}$ defined as $t\ /t_{init}$. The axial translational velocity component of the structures was normalized by its value at the initial time, such that

$\bar{V}_{\omega x}(\bar{t}) = V_{\omega x}(\bar{t}\, t_{init})/V_{\omega x}(t_{init})$. A curve fitting routine was used to compare the experimental data to the model prediction. In the PFI case, the experimental data was approximated as $\bar{V}_{\omega x}(\bar{t}) = \bar{t}^{-2.97}$. For the high-pressure fuel spray, the best curve fit was achieved for $\bar{V}_{\omega x}(\bar{t}) = \bar{t}^{-1.14}$.

The interpretation of these observations was based on the assumption of small vorticity based Reynolds numbers. The vorticity distributions, predicted numerically and analytically for realistic Re look rather different [36]. In the numerical results, the contours expand and shift along the axis of symmetry with time, while forms predicted by the analytical solution only expand during the time interval under consideration. Results of calculations of the time evolution of the vortex ring velocities and energies, based on vorticity distributions, however, predicted by the analytical formulae and numerical solution, show that both variables are not sensitive to Re. Hence, analytical formulae, presented earlier, are expected to be good approximations for realistic values of vortex ring velocities and energies even for relatively high Reynolds numbers. The underlying physics behind these effects was clarified by the newly found analytical solution for the normalized vorticity distribution in the limit of long times in the form $\omega_0 + \mathrm{Re}\,\omega_1$, where ω_0 is the value of vorticity predicted by the classical Phillips solution [29].

Although the experimental results showed some scatter of data, the time evolution of $\bar{V}_{\omega x}$ for the G-DI case showed good agreement with the model that predicts the time evolution of $\bar{V}_{\omega x}$ between $\bar{t}^{-3/2}$ (laminar case) and $\bar{t}^{-3/4}$ (turbulent case). In contrast, the agreement of time dependence of $\bar{V}_{\omega x}$ predicted by the model and observed experimentally for the PFI injector was poor. It is not clear whether this should be attributed to physical processes different from those described by the available models, or to excessive scatter of experimental data.

4.2 The full Lagrangian method and its application to the analysis of vortex ring structures

A new approach to numerical simulation of two-phase flows, based on a combination of the Full Lagrangian method for the dispersed phase [31] and the mesh-free vortex blob method for the carrier phase was suggested. In this case, the problem of calculation of all parameters in both phases (including particle concentration) was reduced to the solution of a high-order system of ordinary differential equations, describing transient processes in both carrier and dispersed phases. This allows one to study in detail local zones of particle accumulation in complex transient flows, including those with multiple intersections of particle trajectories and the formation of 'folds' in the concentration field of the dispersed phase. The main limitation of this approach is that it is applicable to 2D flows only. It was applied to modeling of two processes: the time evolution of a two-phase Lamb vortex and the development of an impulse two-phase jet, leading to the formation of a vortex ring-like structure. These examples involve the formation of local zones of particle accumulation and regions of multiple intersections of particle trajectories. These features of the flow cannot be simulated using the conventional Eulerian and Lagrangian methods described in the literature.

5. Conclusions

Recently developed models, describing the disintegration of liquid jets and the dynamics of vortex ring-like structures at Diesel and gasoline engine-like conditions, are reviewed.

The breakup model for analyzing the evolution of transient fuel sprays characterized by a coherent liquid core emerging from the injection nozzle, throughout the injection process, suggested in [20], is reviewed. The spray breakup is described using a composite model that separately addresses the disintegration of the liquid core into droplets, using the conventional modal approach, and their further aerodynamic breakup. The jet breakup model uses the results of hydrodynamic stability theory to define the breakup length of the jet, and downstream of this point, the spray breakup process is modeled for droplets only. The composite breakup model has been incorporated into the KIVA II Computational Fluid Dynamics (CFD) code and its results are compared with existing breakup models, including the TAB model, stochastic model, the classic WAVE model, the previously developed composite WAVE model (modified WAVE model) and in-house experimental observations of transient Diesel fuel sprays.

The results of comparative analysis of modal and non-modal hydrodynamic stability of round viscous fluid jets are presented following [28]. Both jet fluid and surrounding gas are assumed to be incompressible and Newtonian; the effect of surface capillary pressure is taken into account. The linearized Navier-Stokes equations coupled with boundary conditions at the jet axis, interface and infinity were reduced to a system of four ordinary differential equations for the amplitudes of disturbances in the form of spatial, normal modes. The eigenvalue problem was solved by using the orthonormalization method and the system of least stable normal modes was found. Linear combinations of modes (optimal disturbances) leading to the maximum kinetic energy at a specified set of governing parameters were found. A parametric study of optimal disturbances was carried out for both liquid and air jets in air. For the velocity profiles under consideration, it was found that the non-modal instability mechanism was significant for non-axisymmetric disturbances. The maximum energy of the optimal disturbance was found to be two orders of magnitude larger than that of the single mode.

Following [34, 36], analytical formulae, predicted by recently developed vortex ring models, in the limit of small Reynolds numbers (Re), are compared with the results of numerical solutions of the underlying equation for vorticity and experimental data. Particular attention was focused on the recently developed generalized vortex ring model in which the time evolution of the thickness of the vortex ring core ℓ is approximated as at^b, where a and b are constants ($1/4 \leq b \leq 1/2$) [34]. This model incorporates both the laminar model for $b = 1/2$ and the fully turbulent model for $b = 1/4$. A new solution for the normalized vorticity distribution was found in the form $\omega_0 + \mathrm{Re}\, \omega_1$, where ω_0 is the value of normalized vorticity predicted by the classical Phillips solution. This solution shows the correct trends in the redistribution of vorticity due to the Reynolds number effect. It was emphasized that although the structures of vortex rings predicted by analytical formulae (based on the linear approximation and numerical calculations for arbitrary Re) were visibly different for realistic Reynolds numbers, the values of integral characteristics, such as vortex ring translational velocity and energy, predicted by both approaches, turned out to be close. The values of velocities in the region of maximal vorticity, predicted by the generalized vortex ring model, were compared with the results of experimental studies of vortex ring-like structures in gasoline engine-like conditions. High-speed photography and phase Doppler anemometry (PDA) were used to study the fuel sprays. In general, each spray was seen to comprise three distinct periods: an initial, very short, unsteady phase; a longer, quasi-steady period; and a short, exponential decay phase. The location of the region of maximal vorticity of the droplet and gas mixture was used to calculate the temporal evolution of the radial and axial components of the translational velocity of the vortex ring-like structures [35]. Most of the values of axial velocities lied between the theoretically predicted values corresponding to the later stage of vortex ring development between $b = 1/4$ (fully developed turbulence) and $b = 1/2$ (laminar case).

A new approach to numerical simulation of two-phase two-dimensional flows, based on a combination of the Full Lagrangian method for the dispersed phase [31] and the mesh-free vortex blob method for the carrier phase, is discussed. The problem of calculation of all parameters in both phases (including particle concentration) was reduced to the solution of a high-order system of ordinary differential equations, describing transient processes in both carrier and dispersed phases. The new approach was applied to modeling of the time evolution of a two-phase Lamb vortex and the development of vortex ring-like structures in an impulse two-phase jet.

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